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A baseball batting simulation was used to investigate the information used to hit a baseball. Measures of spatial and temporal swing accuracy were used to test whether batters (a) use speed to estimate pitch height, (b) initiate a constant swing duration at a fixed time to contact, (c) are influenced by the history of previous pitches and pitch count, and (d) use rotation direction. Batters were experienced college players. Pitch speed variance led to predictable spatial errors, and spatial accuracy was worse than temporal accuracy. Swing duration was generally variable. The history of the previous 3 pitches and the pitch count had significant effects on accuracy, and performance improved when rotation cues were added. There were significant effects of expertise on hitting strategy.

From the time the first game of baseball was played, players, coaches, and researchers have sought to understand the act of hitting a pitched ball. Whereas batting coaches and players have explored countless ingenious ways to evaluate and improve batting average and power, researchers have faced the difficult challenge of explaining how this incredible feat of perceptual–motor control is achieved.[1] With margins for error of only a few milliseconds and a fraction of an inch and with processing times of less than half a second, baseball batting truly pushes the limits of human performance.

Despite the Herculean effort that has been put into understanding the act of hitting, relatively little is known about which sources of perceptual information hitters actually learn and how this information is used to control the swing. The major weaknesses of much of the previous research on hitting have been methodological. In the typical laboratory experiments used to study hitting, participants make judgments about stimuli presented on a monitor without actually swinging a bat. There is increasing evidence that the cognitive and perceptual processes involved in these passive judgment tasks may be quite different from those used during goal-directed actions (e.g., Milner & Goodale, 1995). Furthermore, passive judgment experiments cannot show how the information is used to control the motor response. The other major approach researchers have taken is to study real batting (e.g., during batting practice or during use of a pitching machine). These active tasks have the advantage that both perception and action are involved. However, it is often difficult to have fine control over the stimulus parameters and to isolate different sources of information in these real-world settings.

In the present study, I used a novel virtual baseball batting simulation to overcome some of the limitations of previous research. This simulation had the advantage that active motor responses could be combined with fine control over the visual stimulus. Subsequently, I review the previous research on baseball batting to give a background for the specific issues that were addressed in this study.

Perceptual Information Available for Hitting a Baseball

The informational support for hitting a baseball has been explored in some detail (reviewed in Regan, 1997). It has been proposed that the perceptual component of the act of hitting can be reduced to the judgments of *where* and *when*; a batter needs only to know the position of the ball when it crosses the plate and the instant in time that it will be there (Bahill & Karnavas, 1993).

There are two primary sources of visual information about when the ball will cross the plate (the time to contact; TTC). TTC information is provided by the change in angular size of the ball's retinal image, θ (Hoyle, 1957), that is, when an approaching object is moving at a constant speed directly toward the observer's eye[2]:

$$\mathrm{TTC} \approx \frac{\theta}{(d\theta/dt)}. \tag{1}$$

I thank Kristen Macuga for her invaluable assistance in recruiting participants and collecting data for some of these experiments. In addition, I thank Mike McBeath for his many insightful comments on a draft of this article.

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[1] Some of the more interesting methods developed to improve hitting performance include writing numbers on tennis balls and then trying to swing only at odd-numbered balls (used by Barry Bonds; Savage, 1997), trying to hit a Wiffle ball off a tee so that it travels with no spin (used by Tony Gwynn; Gwynn, 1990), and swinging a bat at a telephone pole to notice if one is rolling his or her wrists (used by Ted Williams; Williams & Underwood, 1970).

[2] Of course, the direct approach assumption is not true in the game of baseball, as the point of contact is a considerable distance from the batter's eye. However, Equation 1 does provide a TTC estimate within the temporal margin for error in this situation. If one assumes that the batter makes his or her estimate when the actual TTC is 0.3 s and assumes that the point of contact is 1 m from the eye, the estimation error would be approximately 3.5 ms for a 70-mph (31.3-m/s) pitch and 2.1 ms for a 90-mph (40.2-m/s) pitch (see Tresilian, 1991, for derivation). Both of these errors are well within the ±9-ms temporal margin for error.

It has recently been demonstrated that accurate TTC information is also provided by the change in the binocular disparity (δ) of the approaching ball (Gray & Regan, 1998):

$$\mathrm{TTC} \approx \frac{I}{D(d\delta/dt)}, \quad (2)$$

where D is the distance of the ball from the eye, and I is the interpupillary separation.

The use of the information expressed in Equation 1, commonly called tau (τ) after Lee (1976), has been studied for a wide range of actions (reviewed by Regan & Gray, 2000). For the act of hitting a baseball, Bahill and Karnavas (1993) calculated the value of $d\theta/dt$ to be roughly 30 times above the discrimination threshold at the moment the ball is released, leading to the conclusion that "from the instant the ball leaves the pitcher's hand, the batter's retinal image contains accurate cues for time to contact" (p. 6). As evidence for this claim, they note that batters rarely make purely temporal errors that result in line drives hit into foul territory. However, the ability to make fine discriminations of an information source is necessary but not sufficient for accurate estimation of an absolute value.[3]

In contrast to Equation 1, the approaching ball's distance enters into Equation 2. This may limit its effectiveness, because there are no reliable retinal image correlates of the ball's absolute distance in the hitting situation (Bahill & Karnavas, 1993; Regan, 1997). However, because a baseball is always the same physical size, Equation 2 can be rewritten as

$$\mathrm{TTC} \approx \frac{I\theta}{B(d\delta/dt)}, \quad (3)$$

where B is the diameter of the ball.

For the question of whether a hitter can use visual information to make estimates of absolute TTC with the accuracy required to hit a baseball, one should consider the psychophysical findings of Gray and Regan (1998). In that study, the accuracy of observers' estimates of absolute TTC for a simulated approaching ball was measured over a range of TTC values (from 1.8 to 3.2 s). They reported 2.0%–12.0% errors for judgments based on Equation 1 alone, 2.5%–10.0% errors based on Equation 2 alone, and 1.3%–3.0% errors when both sources of information were available. If one assumes that these percentage values can be generalized from a TTC value of 1.8 s (a 23-mph [10.3-m/s] pitch) to the 0.4–0.6 s TTC range involved in hitting, then a 1.3% estimation error corresponds to a temporal error of approximately 5 ms. This value is well within the ±9-ms error margin calculated by Watts and Bahill (1990). The findings of Gray and Regan suggested that a hitter could estimate the TTC of an approaching ball more accurately if both changing size and changing disparity information are used (although the best estimation performance for either cue alone is also within the required margin for error).

Bahill and Karnavas (1993) have proposed that the more difficult judgment for the batter (and the one with the smaller margin for error; Watts & Bahill, 1990) is estimating where the ball will be when it crosses the plate.[4] The aspect of this judgment that is particularly difficult is predicting how far the ball will drop in height. Although batters are exquisitely sensitive to the angular drop speed of the ball (Regan & Kaushal, 1994), and although this information is well above threshold from the instant the ball leaves the pitcher's hand (Bahill & Karnavas, 1993), the angular drop speed is insufficient for judging height, because the relationship between the angular drop speed and the physical drop speed depends on the ball's distance. In the absence of cues to the ball's absolute distance, two possible means of scaling the angular drop speed with distance to get an accurate estimate of height have been identified. Bahill and Karnavas have proposed that hitters use the pitch speed in lieu of distance information to estimate the height of the ball. In particular, the height of the ball when it crosses the plate (Z_P) is given by

$$Z_P \approx (D_M - tS)(d\phi/dt)\tau, \quad (4)$$

where D_M is the distance to the mound, t is the time since pitch release, S is the estimated pitch speed, and $d\phi/dt$ is the angular drop speed.

Alternatively, Bootsma and Peper (1992) have suggested that batters could take advantage of the fact that the ball is always the same physical size. In particular, the ball height is given by

$$Z_P \approx \frac{B(d\phi/dt)}{d\theta/dt}, \quad (5)$$

where B is the diameter of the ball (for similar derivations, see Regan & Kaushal, 1994; Todd, 1981).

What evidence is there for the use of these information sources in hitting? Bahill and Karnavas (1993; following McBeath, 1990) argue that the use of Equation 4 to estimate height is evidenced by a perceptual illusion that is occasionally experienced by batters: the rising fastball. If a batter underestimates the speed of a pitch, the height estimate based on Equation 4 will be too low. Therefore, at the point of contact, the ball will appear to jump over the hitter's bat. However, Profitt and Kaiser (1995) have argued that hitting strategies requiring an estimate of pitch speed would not produce the temporal precision exhibited by professional hitters.

As support for the use of Equation 5 in estimating height, Bootsma and Peper (1992) cited evidence that introducing balls of different sizes alters the judged spatial position of objects in the horizontal plane. Equation 5 can be used to estimate the lateral distance at which an approaching object will pass the midpoint (by using the angular speed in the horizontal meridian). Using real approaching objects, Bootsma and Peper found that the passing

[3] For example, consider the case of a batter judging the TTC of two pitches, a 95-mph (42.5-m/s) fastball with a TTC of 0.43 s and an 85-mph (38.0-m/s) curveball with a TTC of 0.48 s. If the batter judged that the 95-mph pitch would arrive in 0.6 s and that the 85-mph pitch would arrive in 0.65 s, discrimination of relative TTC would be precise (i.e., the hitter would correctly judge that the 95-mph pitch would arrive sooner, with only a 12% difference in TTC between the pitches), but estimation of absolute TTC would be quite inaccurate (an error of 0.17 s is twice the temporal accuracy required).

[4] Watts and Bahill (1990) calculated the temporal margin for error to be ±9 ms and the spatial margin for error in the vertical dimension to be ±0.5 in. (1.27 cm). Relative to a 95-mph (42.5-m/s) pitch, a 90-mph (40.2-m/s) pitch arrives 21 ms later (i.e., 2.3 times the temporal margin for error) and crosses the plate 2.8 in. (7.11 cm) higher (i.e., 5.6 times the spatial margin for error).

distance at which participants judged an approaching ball to be reachable increased with ball size, as predicted by Equation 5. To my knowledge, this study has not be replicated for judgments of height.

In addition, there have been no investigations of observers' ability to use the relation specified by Equations 4 or 5 to estimate the absolute height of an approaching object. However, Regan and Kaushal (1994) have shown that the discrimination threshold for judging direction on the basis of Equation 5 can be as small as 0.03°–0.12°.

In summary, there are multiple sources of perceptual information available for a batter when estimating where a pitch will be and when it will arrive. The question of which particular sources are used by baseball batters and in what combination remains largely unresolved. In the present study, I directly tested whether batters use speed to estimate pitch height and evaluated whether Equation 1 (i.e., tau) alone is sufficient to control the timing of a swing. I next review the literature on the motor responses involved in hitting.

Biomechanics of Hitting a Baseball

Investigations of the coordinated movements involved in a baseball swing have revealed that hitting involves a complex chain of muscle activity (Shaffer, Jobe, Pink, & Perry, 1993; Welch, Banks, Cook, & Draovitch, 1995). Figure 1 illustrates the phases of a baseball swing. Hitters generate bat speed by a coiling process that involves a rotation of the arms, shoulders, and hips away from the oncoming pitch (the windup). This coiling begins when the batter's weight is shifted toward the back leg by lifting the front foot off the ground. In generating an effective swing, it is important that the hip rotation leads the shoulder rotation, which in turn leads the arm rotation, forming a kinetic link. As the hitter drives forward out of the coil (toward the oncoming ball), his or her foot returns contact with the ground (the preswing). The momentum of the uncoiling of the hips and shoulders is then transferred to the arms (by decelerating each of these components in turn) to create a maximum bat speed (up to 70 mph [31.3 m/s], measured from the end of the bat) at the bottom of the swing (midswing). The preswing begins 175 ms prior to bat–ball contact, and the maximum bat speed occurs approximately 15 ms before contact. It is clear that generating a powerful swing involves much more than just using the arms, as "skilled baseball batting relies on a coordinated transfer of muscle activity from the lower extremities to the trunk, and finally to the upper extremity" (Welch et al., 1995, p. 293). In the present study, I explored how perceptual information is used to modulate some of these swing components.

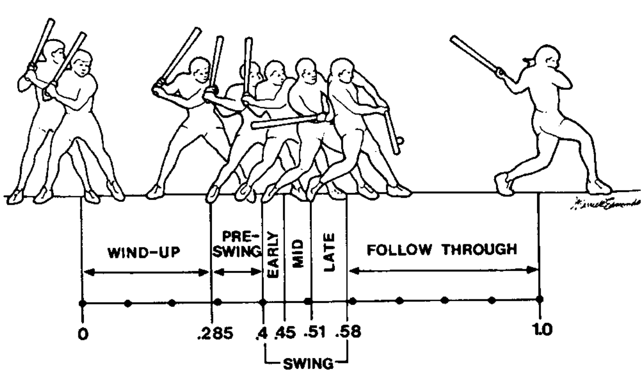

Figure 1. Phases of a baseball swing. Values are proportions of the total swing time. From "Baseball Batting: An Electromyographic Study," by B. Shaffer, F. W. Jobe, M. Pink, and J. Perry, 1993, *Clinical Orthopaedics and Related Research, 292,* p. 286. Copyright 1993 by Lippincott Williams & Wilkins. Reprinted with permission.

Another interesting aspect of the motor act of hitting a baseball is how batters use eye and head movements to track the ball. Hubbard and Seng (1954) were the first to study the visual tracking used by baseball batters. In their study, 35-mm films of professional batters were visually inspected to determine at what intervals during the ball's flight gross eye and head movements occurred. The major finding was that the hitters' smooth pursuit eye movements do not continue until the point of contact: No movements were observed within roughly 150–200 ms prior to contact. This finding is not surprising given that major league pitches can travel at rates up to 1,000 degrees/s, whereas the fastest pursuit eye movements recorded in humans are only about 90 degrees/s (Watts & Bahill, 1990). Hubbard and Seng also somewhat surprisingly found that batters do not seem to reduce this large discrepancy by moving their heads with the flight of the ball.

A more fine-grained analysis, using modern eye- and head-movement recording techniques, was conducted by Bahill and LaRitz (1984). This study compared movements made by Brian Harper, a major league player, with those of novice hitters. The main finding confirmed the observations of Hubbard and Seng (1954): Major league hitters cannot track from release until contact. Harper could track the ball very well until it was roughly 5.5 ft (1.68 m) from the plate, at which point it was no longer foveated. As might be expected, novice hitters lost foveation of the ball considerably earlier, when it was an average of 9 ft from the plate. Harper also used a superior strategy for following the ball: He used a combination of head and eye movements to follow the ball, whereas amateurs tended to predominately move one or the other.

In summary, a baseball swing comprises a complex series of muscle activations involving several different muscle groups. The response complexity involved in hitting may bring into question how well laboratory research on judgments of TTC extends to realistic game situations. For example, in the experiments demonstrating that observers can accurately estimate TTC on the basis of Equation 1 alone, participants viewed the approaching object while seated with their chins in a headrest (Gray & Regan, 1998). Can people still use this information source (Equation 1) effectively when they are moving their heads, eyes, and limbs? In addition, when trying to hit a ball, there are severe demands on when the estimate of the TTC can be made. In the present study, I evaluate the use of Equation 1 in a more realistic active paradigm.

Previous research has also shown that batters' eye movements are not fast enough to keep the ball on the fovea from the point of release to bat–ball contact and that there appears to be some clear expert–novice difference in hitting strategy. The results of the present study provide evidence for further expertise effects in hitting.

Mutual Constraints of Perception and Action

Unfortunately, very few studies have investigated the specifics of *how* the perceptual information is used to control the various motor responses involved in hitting. It is clear that there is more involved in hitting than judging where and when; the batter needs to use this information to modify the complex stages of swinging a bat. In addition, the motor responses (in particular, the limitations of eye movements) may constrain the type of information that can be used.

Hubbard and Seng (1954) were the first to note the necessity of this combined analysis: “The sensory and response aspects may be separated for analysis, but in the batting situation the batter’s step and wind-up for the swing must occur while the sensory–perceptual process is under way” (p. 42). Along with the important findings concerning eye movements, this study identified some important relationships between the movements of the swing and the TTC of the pitch. Figure 2A shows the preswing initiation time (defined as the point in time when the batter’s front foot broke contact with the ground) as a function of pitch speed. It is clear from this figure that the timing of the swing initiation varied with pitch speed. Figure 2B plots the TTC at which the preswing was initiated (i.e., the ball’s TTC when the batter lifted his front foot off the ground) as a function of speed. It is evident that the preswing occurred at roughly a constant TTC for all pitch speeds, resulting in a constant swing duration (time between preswing and contact). This finding has important implications for understanding how batters use visual information during hitting. As suggested by Fitch and Turvey (1978), the timing of constant swing duration could be easily controlled by gearing initiation to a critical value of TTC specified by Equation 1 and/or Equation 2. However, Profitt and Kaiser (1995) have argued that a constant swing duration could also simply be a consequence of batters attempting to achieve maximum bat speed (i.e., swinging the bat as hard as possible over the same distance would always produce roughly the same duration). The use of this control strategy is tested formally in the present study.

Bahill and LaRitz (1984) explored the relationship between the perceptual information available for hitting and the use of eye and head movements. They found that hitters appear to use two different tracking strategies. Figures 3A and 3B show records of the position of the ball and the position of a professional baseball player’s eyes for two swings. The first tracking strategy (shown in Figure 3A, top) is to use smooth pursuit eye movements in combination with head movements to foveate the ball for as much of its flight as possible. The consequence of this strategy is that the ball will be off the fovea (by up to 35°) for the last 5–6 ft (1.5–1.8 m) of its flight. The alternative strategy (shown in Figure 3B) is to follow the ball with eye movements for the first part of its flight (until it is roughly 25 ft [7.6 m] from the plate) and then make a quick saccade to a point in space that is predicted to be just ahead of the ball. After the saccade is finished, the ball is foveated and tracked with smooth pursuit eye movements until contact is made. There are advantages to each of these strategies. Bahill and LaRitz termed the strategy shown in Figure 3A the *optimal hitting strat-*

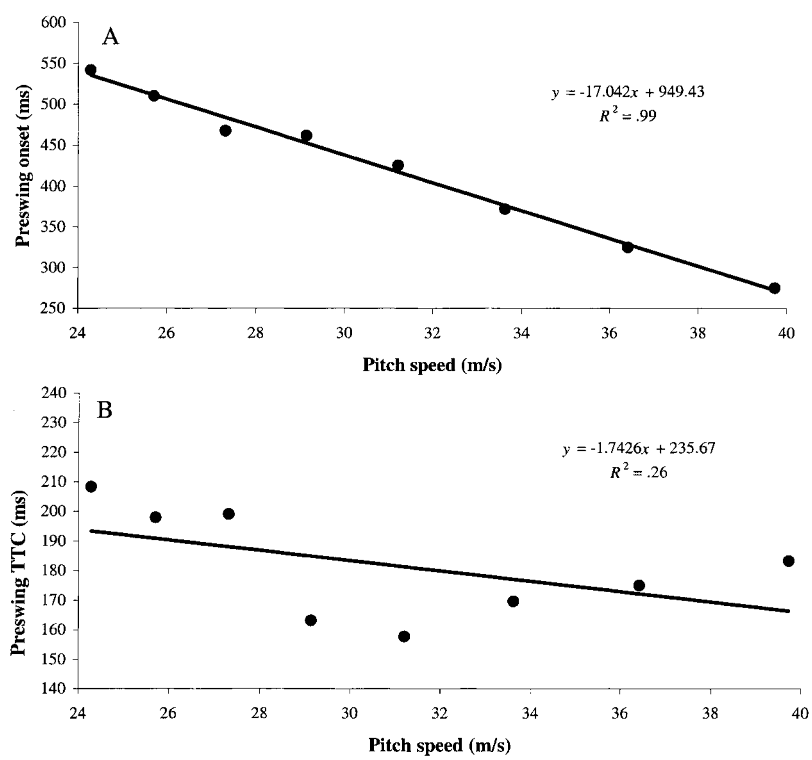

Figure 2. A: Preswing initiation time as a function of pitch speed. B: Preswing time to contact (TTC) as a function of pitch speed. (Data from Table 1 of Hubbard & Seng, 1954, were used to calculate these variables.)

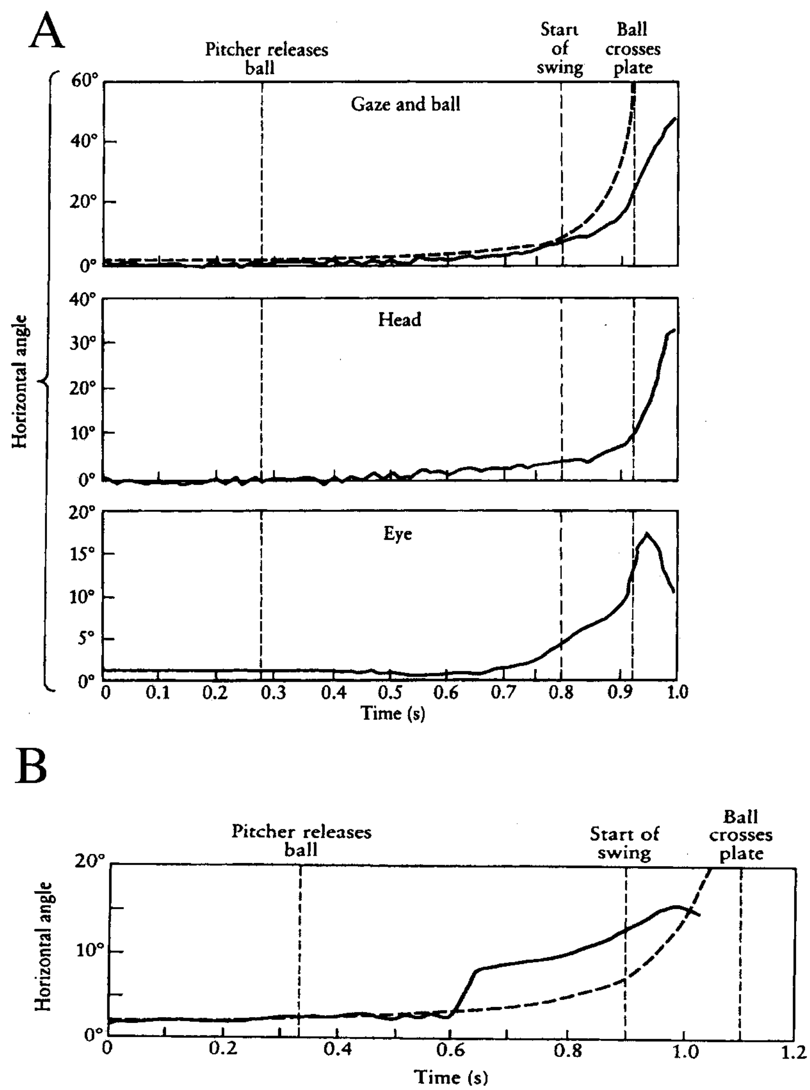

Figure 3. Eye and head movement recordings during a professional baseball player's swing. A: Optimal hitting strategy. The batter follows the ball with his eyes until it is roughly 5–6 ft (1.5–1.8 m) in front of the plate. B: Optimal learning strategy. The batter follows the ball until it is roughly 25 ft (7.6 m) in front of the plate and then makes a saccade so that the ball is on the fovea at the point of contact. From "Why Can't Batters Keep Their Eyes on the Ball?" by A. T. Bahill and T. LaRitz, 1984, *American Scientist, 72,* p. 251. Copyright 1984 by Sigma Xi, The Scientific Research Society. Reprinted with permission.

egy, because it gives the hitter more time to judge the height and TTC of the pitch. The main advantage of the second strategy is that the batter can evaluate the accuracy of his or her height and TTC estimates when the ball is foveated just before it crosses the plate. For this reason, it has been called the *optimal learning strategy,* because it will probably give the batter the best chance of getting hits in future times at bat.

Another interesting problem that arises when considering the mutual constraints of perception and action is that the accuracy of the perceptual information for hitting depends on when it is used. A batter can predict the height of the pitch on the basis of Equation 4 or 5. However, both of these estimates are constant-velocity approximations (i.e., they give the height of the ball at contact assuming it continues to fall at the current drop speed), whereas a baseball accelerates due to gravity. This approximation has the important consequence that the accuracy of the batter's height estimate depends on the point along the ball's trajectory at which the estimate is made. The open circles in Figure 4 plot the height estimate (i.e., the ball's height when it reaches the contact point) provided by Equation 4 as a function of the time since pitch release (bottom axis) and the distance from the tip of the plate (top axis). When the ball is farther than roughly 15 ft (4.6 m), Equation 4 overestimates the height at contact (solid line), and when the ball is nearer than roughly 10 ft (3.0 m), Equation 4 underestimates the

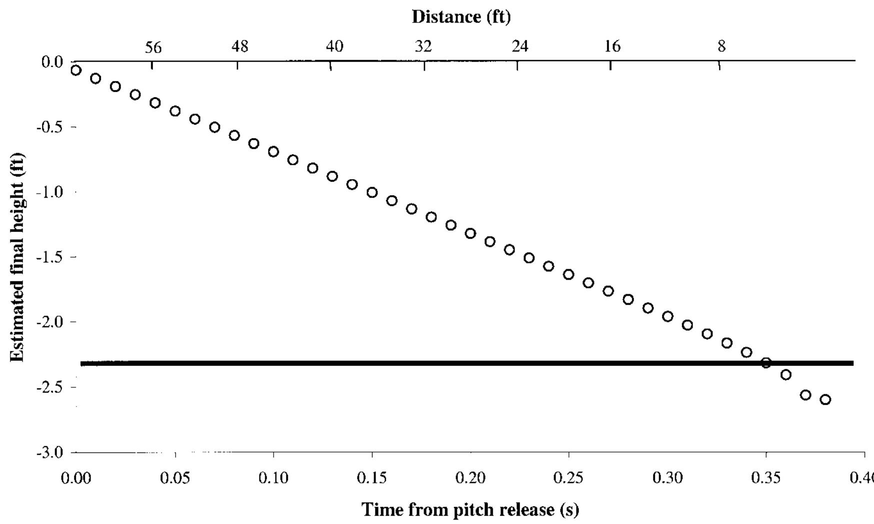

Figure 4. Estimate of final pitch height derived from Equation 4 as a function of time since release and distance from the tip of the plate. The solid horizontal line shows the actual final height for a 90-mph (40.2-m/s) pitch. 1 ft = 0.3048 m.

height. The results of the present study provide evidence that the accuracy of a baseball swing is influenced by this particular estimation error.

Cognitive and Indirect Information for Hitting

Ted Williams (a career .344 hitter considered by many to be the greatest hitter ever) once said "Proper thinking is 50 per cent of effective hitting" (Williams & Underwood, 1970, p. 29). There is abundant anecdotal evidence that hitters use variables such as the history of previous pitches and the pitch count (i.e., the number of balls and strikes) to predict the location and speed of the upcoming pitch. However, to my knowledge, the use of this cognitive information during baseball batting has not been explored in detail.[5]

An indirect perceptual cue that baseball batters could use to distinguish between fast and slow pitches is the ball's rotation direction. Because of the biomechanics and physics of each pitch, a fastball travels with an underspin (or backspin; i.e., from ground to sky), whereas a curveball travels with an overspin. In laboratory judgment experiments, it has been demonstrated that college baseball players can distinguish between a fastball and a curveball from a 200-ms video of the ball's flight at a 90% accuracy rate (Burroughs, 1984). In the present study, I test whether batters can use this information source to improve hitting performance.

Aims of the Present Study

The aims of the study reported here were to address some of the outstanding issues from previous research on baseball batting and to evaluate the effectiveness of the virtual batting task as a research tool. Measures of the temporal and spatial swing accuracy were used to test the following specific hypotheses:

1. If batters use pitch speed to estimate height, introducing a large variance in speed should cause large spatial errors in the swing.
2. Batters can effectively control the timing of their swing when only the TTC information expressed in Equation 1 is available.
3. Batters use a control strategy of initiating a constant swing duration when the ball reaches a critical TTC.
4. Swing accuracy is related to the history of previous pitch types.
5. Swing accuracy is related to the pitch count.
6. Batters can use the direction of ball rotation to improve their swing accuracy.

General Method

Participants

A total of 6 experienced college-level baseball players from the Boston area participated in this study. At the time of these experiments, 2 of the batters (1 and 2) were playing in Division 1A (the highest college level), 2 were playing in Division 3 (3 and 4), and 2 were playing in competitive recreational leagues (5 and 6). The mean number of years of playing experience was 15. All were given 30 min of simulated batting practice prior to beginning the experiments.

Apparatus

Participants swung a baseball bat at a simulation of an approaching baseball. The simulated ball, generated using OpenGL (Silicon Graphics, Inc., Mountain View, CA), was an off-white sphere texture mapped with red laces. The background was black. Simulated lighting from above

[5] The term *cognitive information* is used in this article to refer to sources of information that do not provide direct information about the flight of the ball (and that are not given by the motion of the ball) but that are related to characteristics of the ball's flight. For example, a pitch count of 3–0 provides cognitive information that the next pitch will be fast, whereas a small value of Equation 1 provides direct perceptual information that the pitch is fast.

produced shadows on the bottom of the ball. The simulated ball was displayed on a 28-cm (vertical) × 36-cm (horizontal) SVGA monitor (Viewsonic model PT795) that ran at 120 Hz. The monitor was viewed from a distance of 3.5 m in a dimly lit room. A sensation of motion toward the batter was created by increasing the angular size of the ball. The vertical position of the ball on the monitor was changed to simulate the drop of the ball as it approached the batter. Unless otherwise stated, the ball rotated at 200 rpm with simulated underspin (i.e., from ground to sky).[6] All participants reported a compelling impression of motion in depth.

Mounted on the end of the bat (a Louisville Slugger Tee Ball bat; 25 in. [63.5 cm] long) was a sensor from a FASTRAK (Polhemus, Colchester, VT) position tracker.[7] The *x, y, z* position of the end of the bat was recorded at a rate of 120 Hz.

Pitch Simulations

The pitch simulation was based on that used by Bahill and Karnavas (1993). Balls were launched horizontally (i.e., 0°) from a simulated distance of 59.5 ft (18.5 m; i.e., the pitcher released the ball 1 ft in front of the rubber). The only force affecting the flight of the ball was gravity. (The effects of air resistance and spin on the ball's flight were ignored.) The height of the simulated pitch, $Z(t)$, was changed according to

$$Z(t) = -0.5g(t^2), \tag{6}$$

where g is the acceleration of gravity (32 ft/s [9.8 m/s]). In the present study, I used simulated pitch speeds ranging from 60 mph (26.8 m/s) to 87 mph (38.9 m/s). Unless otherwise stated, the ball was released from a height of 6 ft (1.83 m) and traveled across the center of the plate (i.e., the lateral position was not varied).

Procedure

Batters attempted to hit the simulated approaching ball. No instructions were given as to how hard to swing the bat. A progress bar that indicated the time until pitch release was presented on the monitor 10 s before the ball appeared on the screen. This was used to give a crude simulation of the pitcher's windup. The progress bar disappeared 0.5 s before the pitch was released.

Visual feedback indicating the success of the batter's swing was given as follows. The *x, y,* and *z* coordinates of the bat and ball as a function of time were used to estimate the point of contact on the bat (if there was contact) and bat speed. These variables were then used to approximate the trajectory and speed of the ball leaving the bat. An image of a baseball diamond was presented on the monitor 5 s after the virtual ball crossed the plate. A red line extending from home plate, indicating the trajectory and distance of the ball, was drawn on the diamond. If no contact was made, a large, red *X* was presented to indicate a strike. Text messages were also displayed to indicate foul balls and home runs (i.e., balls estimated to travel further than 300 ft [91.4 m]). In preliminary experiments, some hitters had difficulty determining why their swings were unsuccessful (e.g., they were too late or too high). Therefore, text messages were displayed in the simulation to indicate large temporal and spatial errors in the swing (e.g., "way late" and "get the bat down").

The time interval between successive pitches was 20 s. Each block of trials consisted of 20 pitches. At the end of each block, a statistical summary showing the numbers of contacts, fair balls, and home runs was displayed on the monitor. Batting performance was measured in four different experimental conditions. The order of experiments was randomized across the 6 participants to reduce any practice effects.

Data Analysis

Figure 5 shows a typical recording of bat and ball heights as a function of time. To explore the perceptual–motor strategies used in the simulated hitting task, I calculated the following variables for each swing: the minimum bat height, which was the minimum *Z* value of the bat during the swing (A in Figure 5); and the onset of the downward motion of the bat, which was the point in time that the bat began moving downward from the batter's shoulder (B in Figure 5). The criterion for the latter variable was five consecutive height samples that were lower than the previous sample; this variable is equivalent to the preswing in Figure 1.

When hitting a baseball, the spatial and temporal components of the swing are tightly coupled; for example, for the ball to be contacted earlier in its flight, the bat must be swung higher. To dissociate the spatial and temporal components of the swing, I made the following simplifying assumptions: (a) the batter uses visual information about the height of the ball to control the minimum bat height, (b) the batter uses visual information about the TTC to control the point in time when minimum bat height is reached, and (c) the batter attempts to make contact with the ball when it is 0.9 m in front of the plate (Bahilll & Karnavas, 1993). From these assumptions, the spatial swing error can be expressed as

$$Z\_ball_{y=0.9} - \min(Z\_bat), \tag{7}$$

where $Z\_ball_{y=0.9}$ is the height of the ball when it was 0.9 m in front of the plate, and min(Z_bat) is the minimum bat height. The temporal swing error is given by

$$t\_ball_{y=0.9} - t\_bat_{\min(Z)}, \tag{8}$$

where $t\_ball_{y=0.9}$ is the time (measured from the point of release) when the ball was 0.9 m in front of the plate, and $t\_bat_{\min(Z)}$ is the point in time when the minimum bat height occurred.

Experiment 1: Batting for a Wide Range of Pitch Speeds

The purpose of Experiment 1 was threefold. First, I sought to evaluate batting behavior in a virtual simulation. Given that accurate information about TTC and pitch height was available in the simulation, I predicted that the batters would be able to control the dynamics of their swings appropriately for the simulated pitches. Second, I sought to test the hypothesis that large variances in pitch speed lead to spatial errors in the swing (Bahill & Karnavas, 1993; McBeath, 1990). Third, I sought to test the proposal that batters use a motor control strategy of initiating a constant swing duration at a critical TTC (Fitch & Turvey, 1978).

Method

In Experiment 1, I examined batting performance when pitch speed was chosen randomly on each trial from a wide range of speeds (63–80 mph [28.2–35.8 m/s]). Each participant completed three blocks of 20 swings. There was a 5-min rest period between each block.

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[6] The ball spin rate of 200 rpm was chosen so that it would be perceptible to the batters given the 120-Hz frame rate of the display and to avoid any aliasing. This rate is well below the rotation rate of a major league fastball (roughly 1,600 rpm; Watts & Bahill, 1990) and is closer to the range of a knuckleball (25–50 rpm).

[7] All batters reported that the position tracker in no way hindered their swing and that swinging the bat felt "easy and natural." It should be noted, however, that the bat used in the present study was lighter and shorter than bats typically used in college hardball. Given that bat size can have a dramatic effect on the dynamics of a swing (Watts & Bahill, 2000, p. 109), it will be interesting for future research to address whether bat size also affects the hitting strategy.

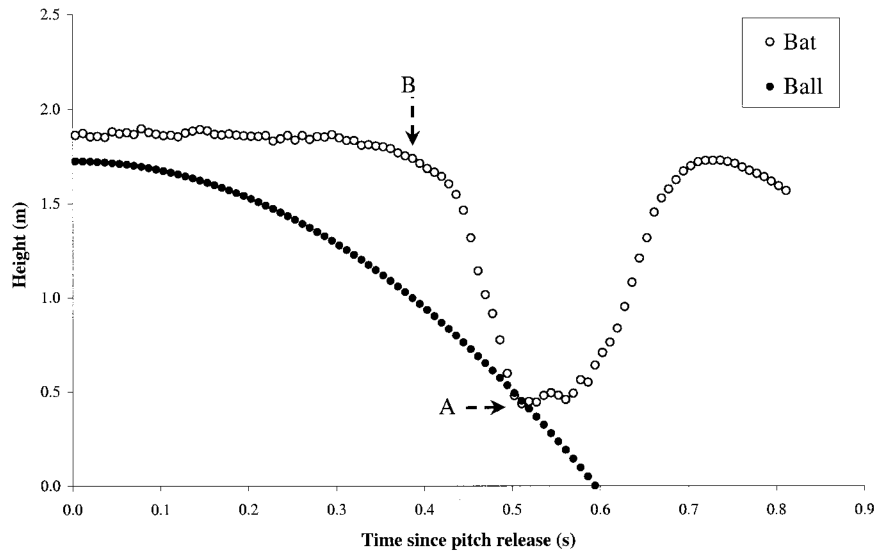

Figure 5. Example data record for one swing. Open circles represent the bat height as a function of time since release, and solid circles represent the ball height. Two main variables were calculated for each plot: minimum bat height (A) and onset of the downward motion of the bat (B).

Results

Overall performance. The most basic requirement for performing the simulated hitting task was to swing the bat at different heights and at different times depending on the pitch speed. Figure 6A shows the minimum bat height [min(Z_bat)] as a function of pitch speed for Batter 3.

As can be seen from the figure, the minimum bat height significantly correlated with pitch speed ($R = .60$, $p < .001$). However, the variation in swing height (slope $= 0.03$ m) was much less than the actual variation in pitch height (thin line; slope $= 0.10$ m). Variability was also quite large for this batter. Similar results were obtained for the other 5 batters. The slope and correlation values for minimum bat height as a function of pitch speed for the 6 batters are shown on the left side of Table 1. Speed and minimum bat height were significantly correlated for all 6 batters, and the slope values ranged from 26%–57% of the actual pitch height slope.

Figure 6B plots the point in time since release when the minimum bat height [$t\_bat_{min(Z)}$] occurred as a function of pitch speed for Batter 3. There was a significant negative correlation between speed and $t\_bat_{min(Z)}$ for this batter ($R = -.43$, $p < .001$). The slope of the line of best fit (-11.25 ms; solid line) was much closer to the predicted slope (-17.90 ms; thin line) in this component than it was in the case of the spatial component (Figure 6A), indicating that this batter was much better at controlling the timing of his swing than the swing height. The slope and correlation values for time of the minimum bat height as a function of pitch speed for the 6 batters are shown on the right side of Table 1. There was a significant negative correlation between speed and $t\_bat_{min(Z)}$ for all 6 batters, and the slope values ranged from 48%–118% of the actual arrival time slope.

For the 60 swings shown in Figure 6, 10 resulted in some simulated contact with the ball. Of the swings that resulted in

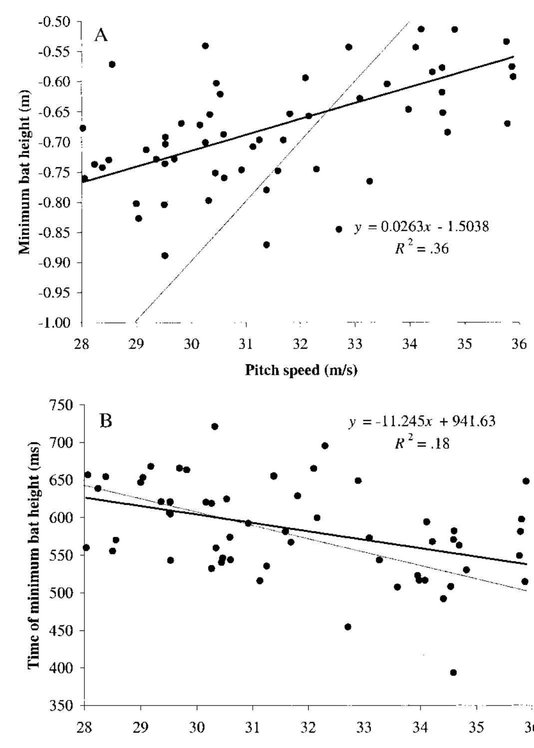

Figure 6. Minimum bat height (A) and time of minimum bat height (B) as a function of speed for Batter 3 in Experiment 1 (random speed simulation). Thick lines are the lines of best fit. The thin line in A is the actual variation in height, and the thin line in B is the time of arrival.

Table 1
Spatial and Temporal Swing Data for Experiment 1

Batter	min(Z_bat) slope	% actual slope[a]	*R*	*p*	$t\_bat_{min(Z)}$ slope	% actual slope	*R*	*p*
1	0.0570	57.0	.54	< .001	−14.10	78.60	−.44	< .001
2	0.0550	55.0	.39	< .01	−21.29	118.60	−.55	< .001
3	0.0260	26.0	.60	< .001	−11.25	62.70	−.43	< .001
4	0.0511	51.1	.63	< .001	−8.77	48.80	−.38	< .01
5	0.0385	38.5	.45	< .001	−10.60	59.08	−.26	< .05
6	0.0287	28.7	.32	< .01	−8.66	48.26	−.39	< .01

Note. min(Z_bat) = minimum bat height; $t\_bat_{min(Z)}$ = point in time when the minimum bat height occurred.
[a] Derived from the height of the simulated ball when it was 0.9 m in front of the plate.

contacts, there were 3 fair balls (none of which were home runs) for this batter. Overall, the success rate of the 6 batters was quite poor; the mean number of hits was 1.7 (SE = 0.3), for a batting average of .030. The mean number of swings that were within the temporal margin for error (M = 12.5, SE = 0.6) was significantly greater than the mean number of swings within the spatial margin for error (M = 5.0, SE = 0.7), $t(5) = -3.3$, $p < .05$. At this point, it should be noted that the hitting task in Experiment 1 was in many ways more difficult than that faced by a major league player; from the comments made by professional hitters (e.g., Williams & Underwood, 1970), it would seem unlikely that a pitcher can randomly vary the pitch speed by 17 mph (7.6 m/s) from pitch to pitch without giving other cues (e.g., arm motion or ball rotation). In Experiments 2–4, I examined more realistic game situations. Subsequently, I present a more detailed analysis of the Experiment 1 data.

Spatial accuracy. Figure 7A shows the data from Figure 6A replotted as spatial error (i.e., Equation 7) versus pitch speed for Batter 3. Negative errors indicate that the bat was above the ball. It is clear from Figure 7A that spatial accuracy was strongly related to pitch speed (R = .89, $p < .001$); this batter swung over the ball at slow speeds and under the ball at fast speeds. What might cause a batter to exhibit this pattern of errors? As shown in Figure 4, the accuracy of the perceptual correlates that can be used to estimate pitch height depends on when the estimate is taken. The dashed line in Figure 7A shows the errors that would be predicted if the batter always estimated pitch height a constant time after the pitcher released the ball. This constant time fit (using a value of 440 ms) gave a reasonably good fit to the data (R^2 = .49). As shown in Table 2, similar results were obtained for the other 5 batters. The mean absolute spatial error for the 6 batters was 0.2 m ($SE < 0.1$), 16 times the spatial margin for error.

Temporal accuracy. Figure 7B shows temporal error (i.e., Equation 8) as a function of pitch speed for Batter 3. Negative errors indicate that the swing was late. The temporal error was not significantly correlated with pitch speed for Batter 3 ($R = -.25$, $p > .05$). Slope and correlation values for all 6 batters are shown in Table 3. For 3 of the 6 batters, there was a significant negative correlation between temporal error and pitch speed. The mean temporal error for the 6 batters was 58.7 ms (SE = 9.8), 6.5 times the margin for error.

Swing initiation and duration. Figure 8A plots swing duration as a function of pitch speed for Batter 3. For this batter, swing duration was significantly shorter at faster pitch speeds (slope = −7.60 ms; $R = -.35$, $p < .01$). This pattern is consistent with the findings of Hubbard and Seng (1954; see Figure 2), although the effect of speed on swing duration was much larger in the present study; the mean slope for the 6 batters was −10.77 ms. Slope and correlation values for all 6 batters are shown in Table 4. Pitch speed and swing duration were significantly correlated for all 6

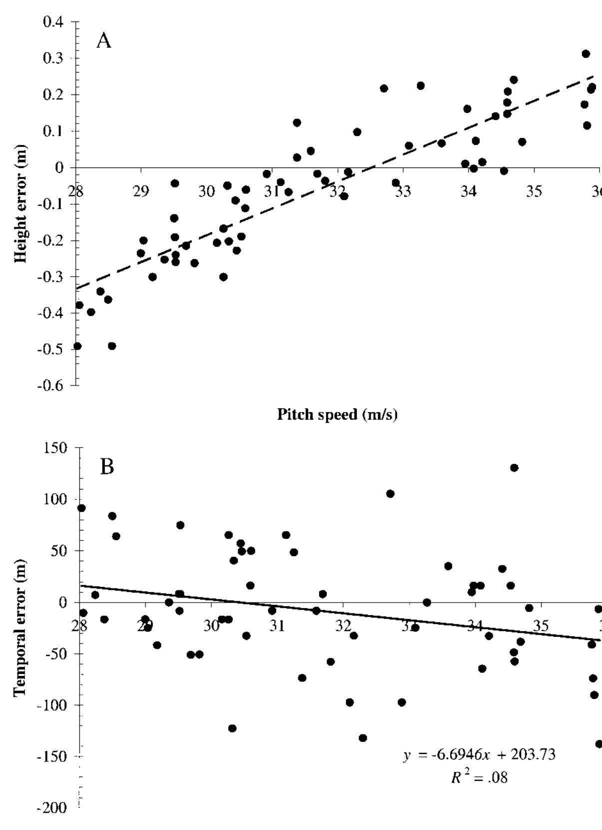

Figure 7. A: Height error as a function of pitch speed for Batter 3 in Experiment 1 (random speed simulation). The dashed line shows values that are predicted if the batter estimated height using a constant-velocity approximation 440 ms after the pitch was released. B: Timing error as a function of pitch speed for Batter 3. The solid line is the line of best fit.

Table 2
Spatial Swing Error Slopes and Curve Fits for Experiment 1

Batter	Spatial error slope	*R*	Constant *t* value (curve fit)	R^2
1	0.042	.43	0.540	.21
2	0.060	.48	0.436	.19
3	0.070	.89	0.440	.48
4	0.051	.61	0.435	.36
5	0.060	.61	0.617	.25
6	0.075	.65	0.494	.28

Note. For all *R*s, $p < .001$.

batters. The mean swing duration for the 6 batters was 148 ms ($SE = 13$). This value is similar to that found in the study by Welch et al. (1995), which used electromyography.

Figure 8B plots TTC at swing onset. There was a strong tendency to start the swing at a shorter TTC for faster pitches (slope = −14.30; $R = -.49$, $p < .001$). Again, the effect of speed, although consistent with the general pattern of Hubbard and Seng's (1954) data, was much larger in the present study; the mean slope for the 6 batters was −17.30 ms. Slope and correlation values for all 6 batters are shown in Table 5. There was a significant negative correlation between speed and TTC at swing initiation for all 6 batters. The pattern of data in Figure 8B is consistent with the batter initiating the swing at a roughly constant time after pitch release. For all 6 batters, pitch speed and swing initiation time were not significantly correlated (see the right side of Table 5).

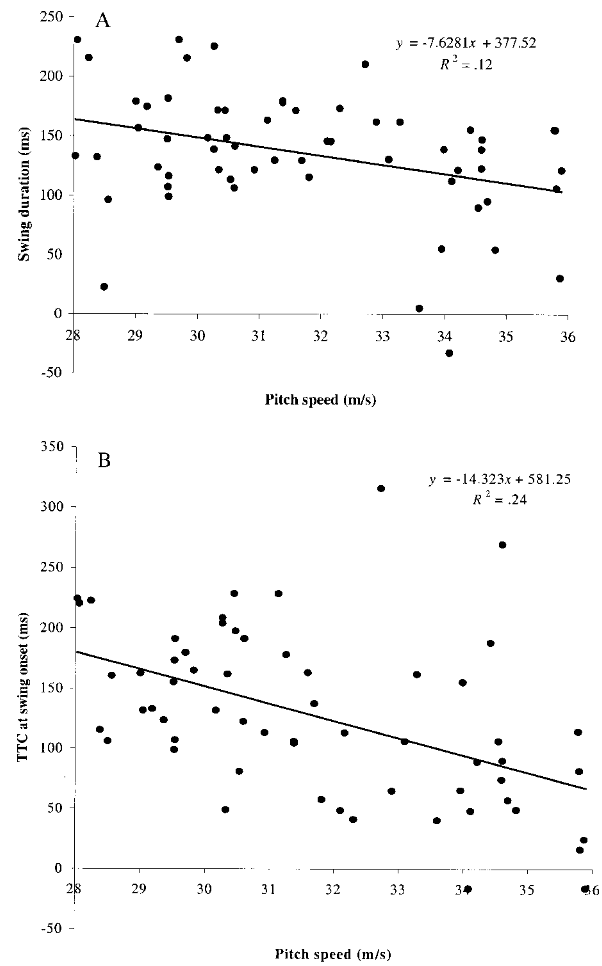

Figure 8. A: Swing duration as a function of pitch speed for Batter 3 in Experiment 1 (random speed simulation). B: Time to contact (TTC) at swing onset as a function of pitch speed for Batter 3. Solid lines are the lines of best fit.

Discussion

Ted Williams (Williams & Underwood, 1970) said that "hitting a baseball . . . is the single most difficult thing to do in sport" (p. 7), and the results of Experiment 1 certainly provide support for his claim. The simulation used in the present study contained theoretically accurate perceptual information about where and when the ball would cross the plate, yet all 6 experienced college players could barely make contact with the ball. It is clear that successful batting is nearly impossible in the situation in which pitch speed is random and in which no auxiliary cues (e.g., pitcher's arm motion or pitch count) are available to the batter.

Relative to the margins for error, batters in the present experiment were significantly better at controlling the temporal component of the swing than the spatial component of the swing. The mean number of swings within the temporal margin for error was significantly greater than the mean number of swings within the spatial margin for error. This experimental finding is consistent with the theoretical analysis by Bahill and Karnavas (1993). This may occur because there is no direct perceptual correlate of ball height; instead, batters must estimate height indirectly using absolute speed or absolute distance. Conversely, TTC can be directly estimated using the ball's rate of expansion. Another problem associated with estimating pitch height is when to take the estimate. The curve fit in Figure 7A suggests that a major source of error in the spatial component of the swing is the batter estimating pitch height at a constant time after the pitch is released.

In general, the results from Experiment 1 are roughly consistent with the following perceptual–motor control strategy: (a) Initiate the swing at a constant time after the pitch is released (Figure 8B), (b) estimate pitch height using a constant-velocity approximation at the point in time that the swing is initiated (Figure 7A), and (c)

Table 3
Temporal Swing Errors for Experiment 1

Batter	Temporal error slope	*R*	*p*
1	−9.1	−.47	< .001
2	4.2	.12	*ns*
3	−6.7	−.25	*ns*
4	−8.6	−.51	< .001
5	−6.1	−.15	*ns*
6	−9.5	−.42	< .01

Table 4
Swing Duration Slopes for Experiment 1

Batter	Duration slope	*R*	*p*
1	−9.71	−.35	< .01
2	−16.63	−.38	< .01
3	−7.62	−.35	< .01
4	−5.80	−.41	< .01
5	−18.95	−.54	< .001
6	−5.91	−.26	< .05

adjust the swing duration based on an estimate of TTC or pitch speed. This control strategy is very different from that proposed by Fitch and Turvey (1978) and is discussed in more detail below.

Experiment 2: Simulation of a Two-Pitch Pitcher

It is a very unusual situation in which a baseball pitcher can randomly vary pitch speed over a 17-mph (7.6-m/s) range. It is more common for a pitcher to have two or three pitches (e.g., a fastball, a curveball, and a change-up) with distinct ranges of speeds. In Experiment 2, I examined the more natural situation in which the simulated pitcher could throw only two pitches (a fastball and a slow ball [i.e., a change-up]), and I measured the effect of pitch type on swing error. I hypothesized that batting performance would be significantly improved in this more natural two-pitch scenario.

Abundant anecdotal evidence exists in support of batters using the sequence of previous pitches to control their swing (e.g., Williams & Underwood, 1970). For example, after seeing several off-speed (i.e., slow) pitches, batters often gear up for a fastball. Thus, I hypothesized that the temporal and spatial errors in Experiment 2 would be related to the pitch type of the previous few pitches.

Method

Slow pitches were simulated to travel at 70 ± 1.5 mph (31.3 ± 0.67 m/s), and *fast* pitches were simulated to travel at 85 ± 1.5 mph (38.0 ± 0.67 m/s). Fast or slow speed was chosen randomly on each trial. As in Experiment 1, all pitches were strikes and traveled down the center of the plate. Each batter completed three blocks of 20 pitches, with rest intervals between each block.

Results

Overall performance. Figure 9A plots the minimum bat height [min(*Z*_bat)] as a function of pitch speed for Batter 3. Compared with the results from Experiment 1 (Figure 6A), this batter was substantially better at controlling the spatial component of the swing in Experiment 2. There was a significant correlation between minimum bat height and pitch speed ($R = .62$, $p < .001$), and the slope of the line of best fit (0.08) was much closer to the actual pitch height slope. Similar results were obtained for the other 5 batters. For all 6 batters, there was a significant positive correlation between min(*Z*_bat) and pitch speed (*R*s ranged from .42 to .71). Slope values for the 6 batters ranged between 67% and 90% of the actual pitch height.

In Figure 9A, it can also seen that the distribution of min(*Z*_bat) values was bimodal for both the slow and the fast pitches. For about seven slow pitches (as indicated by the solid arrow in Figure 9A), Batter 3 swung the bat at a height that would be appropriate for a fast pitch (at roughly −0.5 m), and for about eight fast pitches (as indicated by the dashed arrow in Figure 9A), he swung the bat at a height that would be appropriate for a slow pitch (at roughly −1.0 m).

Figure 9B shows the point in time that the minimum bat height occurred [$t\_bat_{\min(Z)}$] as a function of pitch speed for Batter 3. As was the case in Experiment 1 (Figure 6B), this batter was relatively good at controlling the temporal component of the swing. Minimum bat height was significantly negatively correlated with pitch speed ($R = -.48$, $p < .001$), and the slope of the line of best fit was 53% of the actual arrival time slope. Similar to the spatial swing component, the distribution of $t\_bat_{\min(Z)}$ values was roughly bimodal for both the slow and fast pitches. Similar results were obtained for the other 5 batters. For all 6 batters, there was a significant negative correlation between $t\_bat_{\min(Z)}$ and pitch speed (*R*s ranged from −.39 to −.66). Slope values for the 6 batters ranged between 51% and 99% of the actual pitch height.

For the 60 swings shown in Figure 9, 20 resulted in some contact with the ball. There were 8 fair balls (2 of which were home runs) and 12 foul balls. The mean number of hits for the 6 batters was 7.1 ($SE = 0.2$), for a batting average of .120. A paired *t* test revealed that the number of hits was significantly greater in Experiment 2 than in Experiment 1, $t(5) = 3.9$, $p < .05$. In Experiment 2, the mean number of swings that were within the temporal margin for error ($M = 24.1$, $SE = 0.5$) was not signifi-

Table 5
Swing Initiation Slopes for Experiment 1

Batter	TTC at initiation slope	*R*	*p*	Initiation time slope	*R*	*p*
1	−18.879	−.53	< .001	−1.2	.03	*ns*
2	−12.469	−.25	< .05	−5.3	−.10	*ns*
3	−14.320	−.49	< .001	−3.6	−.13	*ns*
4	−16.503	−.69	< .001	−1.6	−.08	*ns*
5	−26.047	−.48	< .001	−8.4	.16	*ns*
6	−15.490	−.52	< .001	−2.7	−.10	*ns*

Note. TTC = time to contact.

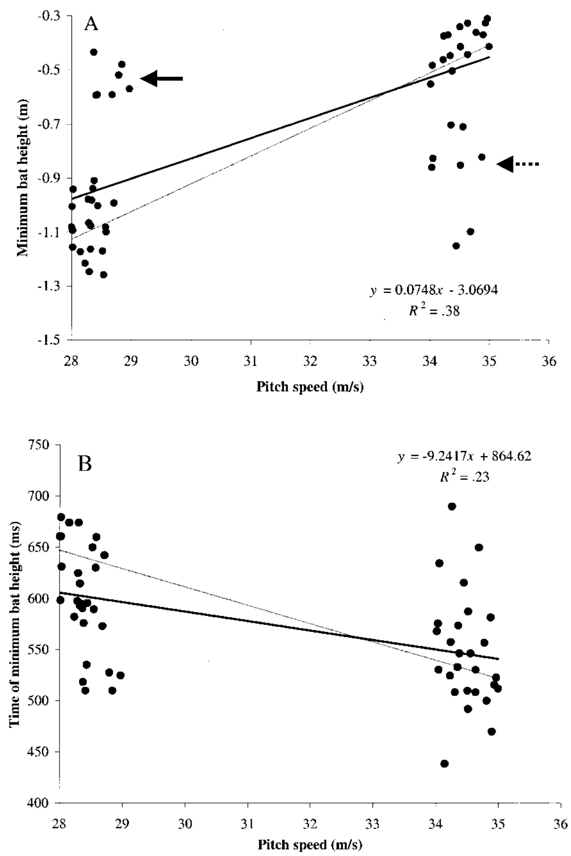

Figure 9. Minimum bat height (A) and time of minimum bat height (B) as a function of speed for Batter 3 in Experiment 2 (two-pitch pitcher simulation). Thick lines are the lines of best fit. The thin line in A is the actual variation in height, and the thin line in B is the time of arrival. Arrows show pitches for which the batter's swing was appropriate for the opposite pitch type: The solid arrow indicates slow pitches for which Batter 3 swung the bat at a height appropriate for a fast pitch, and the dashed arrow indicates fast pitches for which Batter 3 swung the bat at a height appropriate for a slow pitch.

cantly different from the mean number of swings within the spatial margin for error (M = 22.3, SE = 0.7).

Spatial and temporal errors. In Experiment 1, there was a strong relationship between spatial errors and pitch speed, and the pattern of errors was consistent with the batter estimating pitch height at a constant time after release. In Experiment 2, the constant time strategy curve could not be fit to the data because of the limited range of pitch speeds; however, there was a significant positive correlation between spatial error and pitch speed for Batter 3 (R = .29, p < .05). This relationship appeared to be due to Batter 3 being fooled on some pitches: He swung too high for some slow pitches and too low for some fast pitches (see the arrows in Figure 9A). These expectancy effects are discussed in detail below. Similar results were obtained for the other 5 batters. For all 6 batters, there was a significant positive correlation between spatial error and pitch speed (Rs ranged between .21 and .42).

In Experiment 1, the relationship between temporal error and pitch speed was not consistent across the 6 batters. This was not the case in Experiment 2. For all batters, there was a significant negative correlation between speed and temporal error (Rs ranged between −.29 and −.61). Again, this appeared to be due to an expectancy effect, as batters swung early for some slow pitches and late for some fast pitches (see Figure 9B).

Expectancy effects. The bimodal distributions in Figures 9A and 9B suggest that for at least some pitches, Batter 3 generated a swing that was appropriate for the opposite pitch type. Figure 10 plots spatial swing error versus temporal swing error for Batter 3. The relationship in Figure 10 is what would be predicted if the batter's expectations about speed were incorrect on some pitches. For example, if the batter was expecting a fast pitch when the pitch was slow, he should swing too high and early. On the other hand, if he incorrectly expected a slow pitch, he should swing too low and late. There was a strong negative correlation (R = −.87, p < .001) between spatial and temporal errors for Batter 3. Similar results were obtained for the other 5 batters (Rs ranged from −.67 to −.95).

What might batters be using to develop these expectations about pitch speed? One likely possibility is the pitch sequence. For example, if a batter is thrown several pitches of the same speed in a row, he or she might come to expect that the next pitch will also be the same speed. If the batter is using this strategy, then his or her swing error should be much larger when a fast pitch is preceded by a series of slow pitches (i.e., incorrect expectation) than when a series of consecutive fast pitches is followed by a fast pitch (i.e., correct expectation). Figure 11 shows that this was indeed the case. For all 6 batters, the mean spatial (Figure 11A) and temporal (Figure 11B) errors for fast pitches that were pre-

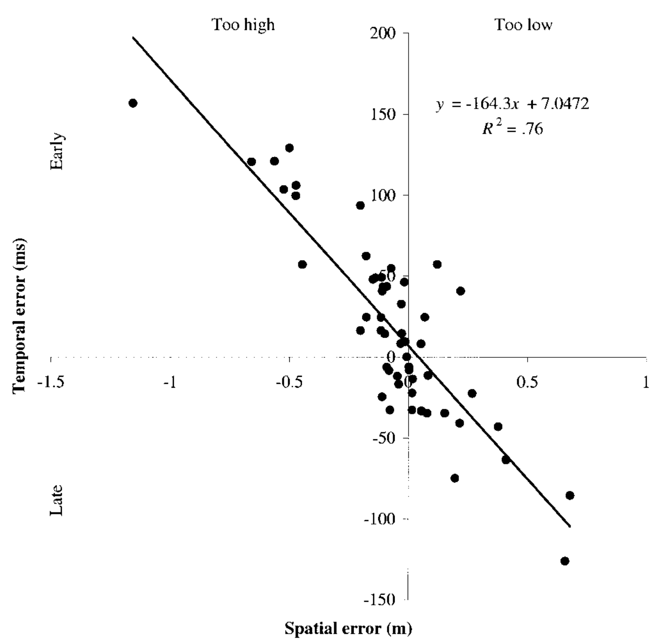

Figure 10. Relationship between temporal errors and spatial errors in Experiment 2 (two-pitch pitcher simulation). Data are for Batter 3. The solid line is the line of best fit.

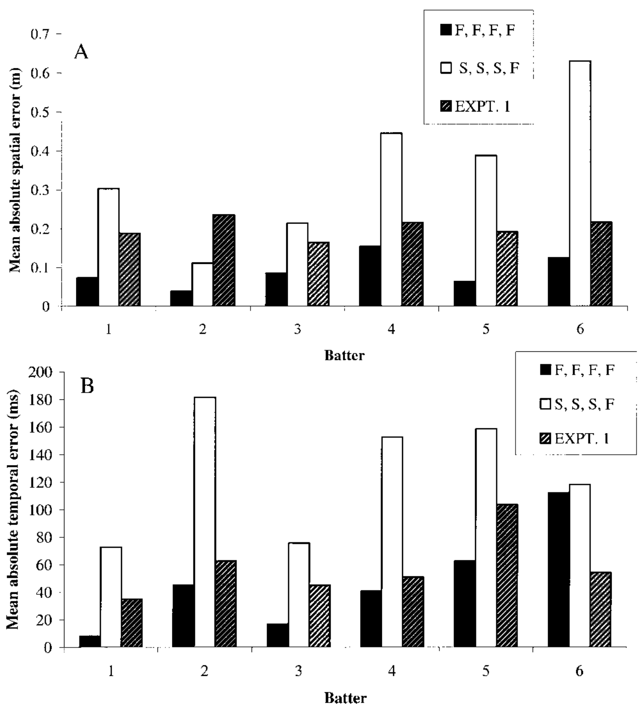

Figure 11. Mean absolute spatial errors (A) and mean absolute temporal errors (B) for the 6 batters in Experiment 2 (two-pitch pitcher simulation). Solid bars are means for fast pitches that were preceded by three consecutive fast pitches (F, F, F, F), open bars are means for fast pitches that were preceded by three consecutive slow pitches (S, S, S, F), and striped bars are means for Experiment 1 (random speed condition). EXPT. = Experiment.

ceded by three consecutive fast pitches (solid bars) were substantially smaller than mean errors for fast pitches that were preceded by three consecutive slow pitches (open bars).[8] Paired *t* tests revealed that the mean errors were significantly different in these two conditions: spatial errors, $t(6) = 2.6$, $p < .05$; temporal errors, $t(6) = 3.1$, $p < .05$. For comparison, the mean errors from Experiment 1 (striped bars) are also plotted in Figure 11. For 5 of the 6 batters, errors were smaller for the random speeds in Experiment 1 than for the unexpected fast pitch in Experiment 2.

In a companion article (Gray, 2002), I used a two-state Markov model to predict expectancy effects for all possible pitch sequences. This mathematical model uses a simple set of transition rules to determine the probability that the batter will expect a given pitch on the basis of the type of the previous three pitches. This probability can then be used to predict the accuracy of the swing (i.e., if there is a high probability that the batter is expecting a fast pitch, he or she will be more accurate when the pitch actually is fast). This model provided a good fit to the swing error data in this article (R^2 values for the 6 batters ranged from .51 to .96).

Discussion

As predicted, hitting performance was considerably better when the simulated pitcher was limited to throwing only fastballs and change-ups. In comparison with Experiment 1, the batters made significantly more contacts with the ball, and control of the spatial component of the swing significantly improved. The dramatic improvement in hitting performance for this two-pitch scenario emphasizes why it is important for baseball pitchers to learn to throw at least three different types of pitches (Ryan & House, 1991).

Why is it easier to hit against a two-pitch pitcher? The results of Experiment 2 indicate that it is because batters can anticipate the speed of the upcoming pitch on the basis of the prior sequence of pitches and use this cognitive information to control their swing. For the 6 batters, the spatial swing error was up to five times larger and the temporal swing error was up to eight times larger when a fast pitch was preceded by three consecutive slow pitches than when it was preceded by three consecutive fast pitches. This dramatic difference in performance occurred even though the perceptual information was identical in the two situations. It is clear that, as I hypothesized, the batters were supplementing the visual information about the current pitch with expectations about speed on the basis of the history of previous pitches.

Despite the improvement in hitting performance in Experiment 2, the batters had very large swing errors on some pitches, and the batting average (.120) was still low relative to real-game performance. The strong relationship between errors and pitch speed shown in Figure 10 suggests that the major source of error in Experiment 2 was incorrect expectations about pitch type. When batters did make a large error, they made it because their swing was appropriate for the opposite pitch speed. Bahill and Karnavas (1993) have proposed that batters need to use supplemental information about pitch speed to estimate pitch height (see Equation 4). Further, these authors predicted that if batters are using Equation 4, then they should swing too low for unexpectedly fast pitches (which give the batter the illusion that the ball has risen) and too high for unexpectedly slow pitches (which give the batter the illusion of a hard-breaking pitch). The pattern of errors shown in Figures 9 and 10 provides strong experimental support for their predictions.

Experiment 3: Effect of Pitch Count

In Experiment 2, I found that swing accuracy was related to the history of previous pitches. It is well-known that baseball batters also generate expectations about the upcoming pitch on the basis of the pitch count: "Certainly the pitch you anticipate when the count is 0 and 2 (a curve ball probably, if the pitcher has one) is not the pitch you anticipate when the count is 2 and 0 (fastball, almost without exception)" (Williams & Underwood, 1970, p. 30).

In Experiment 3, I examined the effect of pitch count on hitting performance for the two-pitch scenario used in Experiment 2. From the narratives from experienced players, I predicted that batters would expect fast pitches when they were ahead in the count (i.e., more balls than strikes) and that they would expect slow pitches when they were behind in the count. The rationale for these predictions is as follows. In general, for slower pitches such

[8] Similar results were obtained when swing errors for slow pitches were analyzed. The mean temporal (and spatial) errors for a slow pitch preceded by three consecutive slow pitches were significantly lower than those for a slow pitch preceded by three consecutive fast pitches.

as a change-up or curveball, it is more difficult for a pitcher to control the location. When the pitcher falls further behind in the count, it is more likely that the next pitch thrown will be fastball to ensure a strike (Williams & Underwood, 1970, p. 30). On the other hand, as the hitter falls behind in the count, it becomes more likely that the pitcher will attempt to force the batter to chase a pitch such as a slow curveball or change-up (Williams & Underwood, 1970, p. 30).

Method

In Experiment 3, the apparatus and procedure used were identical to those in Experiment 2, except that the horizontal locations of the simulated pitches were varied such that some of the pitches were strikes and some of the pitches were balls. For strikes, the horizontal location of the simulated baseball when it crossed the plate was 0.0 ± 1 in. (0 ± 2.54 cm), for which 0.0 was the center of the plate. For balls, the horizontal location of the pitch when it crossed the plate was either −12 ± 1 in. (−30.5 ± 2.54 cm) or +12 ± 1 in. (30.5 ± 2.54 cm). Pitch location was chosen randomly on each trial to be either a ball or strike. Pitch speed on each trial depended on the pitch count (balls to strikes). For counts of 0–0, 1–0, 0–1, 1–1, 2–1, 2–2, and 3–2, fast pitches and slow pitches occurred with equal probability. For counts of 0–2 and 1–2, slow pitches occurred with a probability of .65, and fast pitches occurred with a probability of .35. For counts of 2–0, 3–0, and 3–1, fast pitches occurred with a probability of .65, and slow pitches occurred with a probability of .35. Thus, as is generally the case in real baseball, fast pitches in the simulation occurred with a higher probability when the hitter was ahead in the count, and slow pitches in the simulation occurred with a higher probability when the hitter was behind in the count. If the batter swung the bat at a ball and the bat crossed the front of the plate, the call for that pitch was a strike. Visual feedback was given for the pitch call, total pitch count, walks, and strikeouts.

Results and Discussion

Figure 12A shows mean spatial swing errors for two different pitch counts: when the batter was ahead in the count at 2–0 (open bars) and when the batter was behind in the count at 0–2 (solid bars). The swing errors shown are only for fast pitches.[9] If, as predicted, batters expected fast pitches when they were ahead in the count and slow pitches when they were behind in the count, then swing errors for fast pitches should be much smaller for a count of 2–0 than for a count of 0–2. This was indeed the case. For all 6 batters, the mean spatial swing error for fast pitches was lower for the 2–0 count than for the 0–2 count. A paired *t* test revealed that errors for the two pitch counts were significantly different, $t(5) = 2.5$, $p < .05$.

Similar results were obtained for the temporal swing errors for fast pitches, as shown in Figure 12B. The mean temporal swing error was significantly smaller for the 2–0 count than for the 0–2 count, $t(5) = 2.9$, $p < .05$. When these pitch-count transition rules (i.e., expect a fast pitch when ahead in the count, and expect a slow pitch when behind in the count) were incorporated into the two-state Markov model described above (Gray, 2002), the model provided a good fit to the swing error data for all possible pitch counts (R^2 values for the 6 batters ranged from .59 to .73).

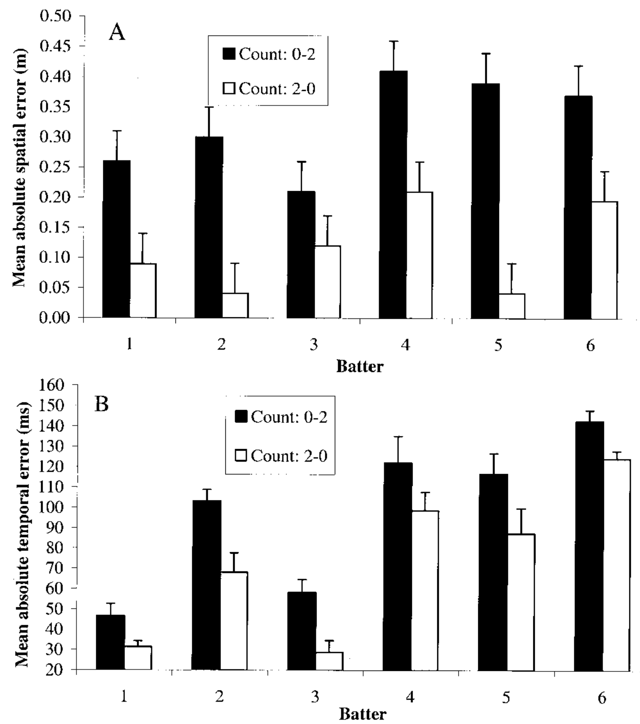

Figure 12. Mean absolute spatial errors (A) and mean absolute temporal errors (B) for the 6 batters in Experiment 3 (pitch-count simulation). Solid bars are means for a count of 0–2 (i.e., no balls and two strikes), and open bars are means for a count of 2–0 (i.e., two balls and no strikes). Error bars represent standard errors.

Experiment 4: Effect of Rotation Cues

The purpose of Experiment 4 was to investigate whether rotation direction cues influence batting performance. Because experienced batters appear to be quite sensitive to this cue (Burroughs, 1984), I predicted that swing errors would be significantly smaller when the rotation direction cue was added to the batting simulation.

Method

The procedure was identical to that described for Experiment 2, except that for fast pitches, underspin was simulated, whereas for slow pitches, overspin was simulated. The spin direction did not otherwise change the trajectory of the simulated pitch; it simply added an additional source of information related to pitch speed. The rotation rate was 200 rpm. All pitches traveled down the center of the plate. Each batter completed three blocks of 20 pitches with rest intervals between each block.

Results

The open bars in Figures 13A and 13B show, respectively, the mean spatial errors and the mean temporal swing errors in Experiment 4. In comparison with the identical two-pitch scenario without rotation cues (Experiment 2; replotted with solid bars), errors were substantially smaller for the majority of the batters. For

[9] Again, the opposite pattern of results was obtained when errors for slow pitches were analyzed: Swing errors (both spatial and temporal) for slow pitches were significantly larger for a count of 2–0 than for a count of 0–2.

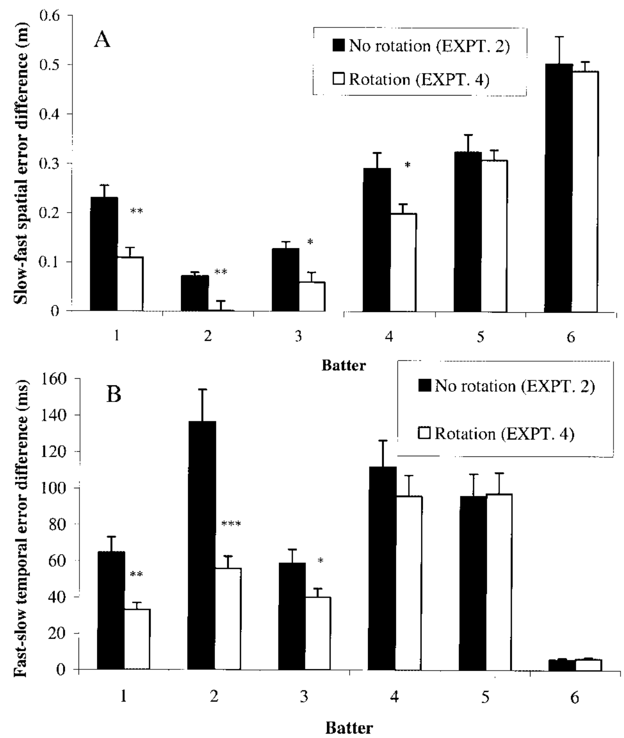

Figure 13. Mean absolute spatial errors (A) and mean absolute temporal errors (B) for the 6 batters in Experiment 4 (rotation cue condition). Solid bars are the means for Experiment 2, in which both slow and fast pitches had the same rotation direction. Open bars are the means for Experiment 4, in which slow pitches had overspin and fast pitches had underspin. Error bars represent standard errors. $*p < .05$. $**p < .01$. $***p < .001$. (Bars without asterisks show nonsignificant results.) EXPT. = Experiment.

Batters 1, 2, 3, and 4, spatial errors were significantly smaller when rotation cues were present. Results of t tests were as follows: Batter 1, $t(118) = 2.0$, $p < .05$; Batter 2, $t(118) = 2.4$, $p < .01$; Batter 3, $t(118) = 2.5$, $p < .01$; Batter 4, $t(118) = 1.8$, $p < .05$. For Batters 1, 2, and 3, temporal errors were significantly smaller when rotation cues were present. Results of t tests were as follows: Batter 1, $t(118) = 1.7$, $p < .05$; Batter 2, $t(118) = 2.5$, $p < .01$; Batter 3, $t(118) = 5.1$, $p < .001$.

Discussion

The results of Experiment 4 indicate that for some batters, rotation direction cues improve hitting performance. This behavioral finding is consistent with psychophysical discrimination findings (Burroughs, 1984). The ability to use rotation cues appears to depend on the hitter's ability to detect the movement of the seams on the ball. Hyllegard (1991) compared hitters' ability to discriminate between video clips of fastballs and curveballs for a ball that was painted white so that it had no seams, for a regulation baseball, and for a ball for which the visibility of the seams was enhanced with thick red stripes. College players achieved 74% correct for the ball with no seams, 81% correct for the regulation ball, and 85% correct for the ball with enhanced seams.

One limitation of the present finding is that the display frame rate necessitated the use of a rotation rate that was well below that which occurs for real fastballs. Whether batters can still use the rotation direction cue when the spin rates are set at their normal level needs to be addressed in future experiments.

Supplemental Analysis: Effect of Playing Level

Method

The experimental results showed that the simulated batting task requires some of same skills involved in playing baseball. I therefore predicted that several of the experimental variables would correlate with playing level. To test this prediction, I grouped batters into one of three categories according to their current playing level: high (College Division 1), medium (College Division 3), or low (college recreational leagues). In the present study, there were two batters in each category (see the General Method section). I performed correlational analyses on the following variables: Experiment 1, spatial and temporal accuracy and total number of ball contacts; Experiment 2, differences in absolute errors for the conditions in which a fastball followed three fast pitches and a fastball followed three slow pitches; Experiment 3, differences in absolute errors when the batter was ahead in the count (2–0) and when the batter was behind in the count (0–2); and Experiment 4, decrease in absolute errors produced by the addition of rotation cues.

Results and Discussion

Table 6 shows the correlation values for the different experimental variables and playing level. As expected, measures of

Table 6
Correlations Between Playing Level and Experimental Variables

Experiment	Variable	Description	R	p
1	Spatial accuracy	The min(Z_bat) slope compared with the actual pitch height slope	.74	$< .05$
	Temporal accuracy	The $t\_bat_{min(Z)}$ slope compared with the actual arrival time slope	.76	$< .05$
	Total number of contacts	Fair balls plus foul balls	.73	$< .05$
2	Expectation effect size	Difference between mean absolute temporal error when a fastball was preceded by three fast pitches and mean absolute temporal error when a fastball was preceded by three slow pitches	−.77	$< .05$
3	Pitch count effect size	Difference between mean absolute temporal error for a pitch count of 2–0 and mean absolute temporal error for a pitch count of 0–2	.07	*ns*
4	Rotation cue effect size	Difference between mean absolute temporal error for Experiment 2 (no rotation cues) and mean absolute temporal error for Experiment 4 (rotation cues)	.86	$< .05$

hitting performance in Experiment 1 were highly correlated with playing level. Batters playing at a higher level had a greater number of ball contacts, and the variation in their swing height and timing was significantly closer to the actual variation in pitch height and time of arrival. For Experiment 2, higher-level players showed less of an effect of pitch sequence than did lower-level players. Finally, the addition of rotation cues improved performance more for higher-level players than for lower-level players. This result is consistent with the finding that college baseball players are significantly better than novices at discriminating between fastballs and curveballs presented in short video clips (Burroughs, 1984).

General Discussion

Virtual Baseball Batting Task

For many years, researchers interested in visually guided action have struggled with the problem of combining fine control over stimulus parameters with realistic, active motor responses. The baseball batting simulation used in the present study may be a step in the right direction toward addressing this issue. In comparison with hitting real baseballs, virtual batting has the advantage that different information sources can be isolated and dissociated (see Rushton & Wann, 1999, for a comparable catching simulation).[10] The participants in this study reliably adjusted the dynamics of their swing based on the parameters of the simulation and, while doing so, exhibited some of the well-known behaviors observed in real baseball. The fine control over parameters such as pitch speed in the simulation allowed direct testing of some questions that were untenable to judgment experiments or measurements of real batting. Finally, the strong correlations between the dependent measures and playing level in the present study indicate that virtual batting may be a useful diagnostic tool for comparing individual hitters and for studying the nature of expertise.

Perceptual–Motor Control Strategies for Hitting a Baseball

It has been repeatedly emphasized that hitting a baseball places severe demands on the perceptual–motor system. A 100-mph (44.7-m/s) major league fastball travels the distance between the mound and the plate in 410 ms. Combining this with the batter's limited ability to track the ball with his or her eyes (Figure 3) and the movement time required to execute the stages of a swing (Figure 1), it is clear that a baseball batter must predict the future location of the ball from visual information provided in the first 150–200 ms of the ball's flight.

Despite the severity of this task, it should be emphasized that accurate predictive visual information is available through direct perceptual variables. The TTC can be predicted from the ball's rate of expansion (Equation 1), and the ball's height can be predicted from ball diameter, rate of expansion, and angular drop speed (Equation 5). Therefore, as proposed by Bootsma and Peper (1992), it is possible to hit successfully entirely on the basis of perceptual information picked up during the ball's flight. However, the results of Experiment 1 in the present study do not support this direct pick-up proposal. When pitch speed was varied randomly from trial to trial, the batters could not consistently make contact with the ball, even though the information provided by Equations 1 and 5 was available. Furthermore, the large improvement in hitting performance in the two-pitch scenario (Experiment 2) and with the addition of rotation cues (Experiment 4) is not compatible with this proposal, as neither of these manipulations affect the values of Equation 1 or 5.

Instead, these results provide support for the indirect perceptual model proposed by Bahill and Karnavas (1993). These authors have proposed that the height of the ball when it crosses the plate is predicted indirectly from an estimate of pitch speed (Equation 4). Further, the authors argue that "the speed estimator probably uses memory and other sensory inputs: some visual, such as the motion of the pitcher's arms and body" (p. 8). In the present study, the pattern of swing errors in Experiments 2 and 3 was consistent with the batter generating expectations about pitch speed on the basis of pitch sequence and pitch count. As predicted by Bahill and Karnavas, when the batters in this study attempted to hit an unexpectedly fast pitch (e.g., when a fast pitch followed a series of slow pitches), they swung the bat too low. The improvement in batting performance resulting from the addition of rotation cues in Experiment 4 also provides support for this indirect model. Rotation direction does not provide any information to the batter about the future location of the ball, so it presumably aids the batter by influencing his or her estimate of pitch speed.

How does the batter actually use this perceptual information to control the complex motor responses involved in swinging a bat? It is surprising that there have been very few models put forth to explain this essential component of "America's Game." One notable exception is the simple control model proposed by Fitch and Turvey (1978). These authors argued that batters could easily control the timing of their swing by initiating a constant-duration ballistic swing at a critical value of TTC. The results of the present study are not consistent with this proposal, as the batters appeared to initiate a variable-duration swing at a constant time after the pitch was released. Further, the results of Experiment 1 suggest that the batters were adjusting their swing duration on the basis of an estimate of pitch speed or an estimate of TTC. It is clear that further research is needed to understand how a baseball swing is controlled. Some possible future experiments are discussed below.

Cognitive Information for Baseball Batting

Anecdotal evidence from players and coaches (e.g., Williams & Underwood, 1970) indicates that cognitive processing (e.g., expectations about the upcoming pitch) plays an important role in successful baseball batting, yet this aspect of hitting has not been investigated in detail. The present study provides the first quantitative experimental evidence that the history of previous pitches and the pitch count significantly influence the spatial and temporal components of a baseball swing. The pattern of errors produced by manipulations of these variables is consistent with the notion that batters generate expectations about the type of the upcoming pitch. Furthermore, the findings reported here indicate that in some

[10] Simulation also has the advantage that impossible pitches (such as rising fastballs or sharp-breaking curveballs) can be used.

instances, these expectations may play a larger role in swing execution than does visual perception. For example, consider the data from Batter 2 in Figure 11B. On average, this batter swung 137 ms later for a fast pitch that was preceded by three slow pitches than for a fast pitch that was preceded by three fast pitches. Given that the actual mean difference in time of arrival between the fast and slow pitches in Experiment 2 was only 105 ms, it would appear that in this instance, the batter completely ignored the perceptual information and controlled his swing entirely on the basis of his expectation about the speed. This finding may come as no surprise to many baseball fans given the regularity with which professional hitters are fooled by slow pitches. Finally, the results of the present study suggest that picking up of indirect perceptual cues, such as ball rotation direction, is also very important for successful batting.

The hitters in this study seemed to use very simple rules for generating expectations about pitch speed. The results of Experiment 2 suggest that hitters use a simple strategy of expecting the pitcher to throw the ball at the same speed as on the previous two or three pitches in the sequence. The results of Experiment 3 are consistent with the batter expecting fast pitches when they are ahead in the count and slow pitches when they are behind in the count. The emphasis placed by the batter on regularities in the pitch sequence emphasizes why it is so important for a pitcher to vary the speed and location of pitches (Ryan & House, 1991). In a companion article, I provided a detailed mathematical model of the effect of expectations on baseball batting (Gray, 2002).

Effects of Skill Level

Comparisons between athletes of different skill levels are one of the most effective techniques for understanding the specific skills that make great athletes. If an elite player has some ability that a novice does not, then it is presumably important to his or her sport. In the present study, I was able to compare the data for players competing at three different levels. The correlational analyses in Table 6 point to some interesting differences that may be related to hitting success. The finding that less experienced batters are more influenced by the prior sequence of pitches may imply that the degree to which expectations are combined with perceptual information is related to skill level (i.e., good hitters are better at tempering their expectations with perceptual cues). Combining this with the finding that more experienced batters appear to make more use of rotation cues, one can make the general conclusion that experienced players use more sources of information when batting. Finally, the finding that basic performance measures (e.g., temporal and spatial swing errors) were strongly correlated with playing level implies that the simulated hitting task used here required some the same skills used in real baseball.

Limitations and Future Research

It is clear that the task used in the present study was a very simplified simulation of a real baseball game and lacks some information sources that may have a significant effect on hitting performance. One such information source may be the pitcher's delivery. It is well-known that a pitcher's body language can be used to anticipate the upcoming pitch. For example, consider this description by elite major league hitter Tony Gywnn: "His arm is less extended than usual when he throws one kind of pitch, his grip on the ball is too visible on another" (Will, 1990, p. 33). Another important limitation of this simulation is that there was no binocular information available. Binocular cues are important for accurate estimates of TTC (Gray & Regan, 1998). In the optimal hitting situation in the present study (Experiment 4), the mean batting average was only .220, suggesting that these missing information sources are indeed important. It should be emphasized that this simulation also lacked some variables that may make the hitting task more difficult. For example, in the present study all pitches were launched at the same angle, such that slower pitches arrived at a lower height than did fast pitches. In real baseball, this is clearly not the case, as pitchers can (and do) throw high change-ups and low fastballs. I also did not simulate the complex effects of air resistance and spin (Adair, 1990) on the ball flight that would make the trajectory of the ball even less predictable from the perceptual information. Finally, simulations of pitches such as curveballs, knuckleballs, and sliders were not used. The simulation parameters used in the present study were chosen to permit for reasonable success in the hitting task while allowing particular information sources to be isolated and tested and to allow for direct comparison with a previous model of hitting (Bahill & Karnavas, 1993). Of course, to fully understand hitting, it will be necessary for the effects of all these variables to be considered. I plan to pursue this systematically in future experiments.

Implications for Visually Guided Actions

Why were the batters using cognitive and indirect perceptual information when direct visual correlates of height and TTC were available? Is it because hitting is an unusually demanding task that is beyond the limits of the perceptual–motor system? Or is this a general, multipurpose strategy for the control of action? The role of cognitive processing in the control of visually guided action is often overlooked. In fact, most experiments in this area are deliberately designed to remove expectancy and memory effects through randomization of conditions and through the use of unfamiliar objects presented out of context. These highly controlled laboratory conditions remove many of the regularities that are available when people perform actions in the real world. For example, in many situations in which people are required to intercept an approaching object (e.g., when playing baseball or football), the object has a familiar size and travels at a predictable speed. Yet in psychophysical experiments on TTC, it is necessary to vary object speed and object size over a large range (e.g., Gray & Regan, 1998). The present findings suggest that it is important to augment previous psychophysical studies with more realistic conditions to determine if these other sources of information are used (see also Tresilian, 1999).

Finally, a review of earlier research on hitting (see the introduction) suggested that baseball batting could be controlled entirely on the basis of direct perceptual variables. Psychophysical experiments have demonstrated that observers are sensitive to optical variables such as rate of expansion and angular drop speed, and theoretical analyses have shown that these variables can be combined to predict height and time of arrival. However, the present findings indicate that a baseball swing is not controlled in this manner. This discrepancy further emphasizes why it is necessary to study perception and action together.

Summary

This study effectively implemented a novel simulation of baseball batting to study the perceptual and cognitive information used during hitting. It was shown that varying the speed from pitch to pitch leads to large errors in the height of the swing and that batters use the history of previous pitches, knowledge of the pitch count, and ball rotation direction to control their swing. These findings are consistent with the proposal that batters use an estimate of speed to predict pitch height (Bahill & Karnavas, 1993) and that this estimate is largely based on expectations generated before the ball is released. Indeed, Ted Williams seems to be correct: Thinking appears to be a major part of effective hitting. Combining perception and action together in this manner facilitated one of the first studies to address the important question of how these information sources are used to control the swing. Contrary to previous findings (Hubbard & Seng, 1954), these batters seemed to vary the duration of their swing from trial to trial, suggesting that this may be a swing parameter that is controlled on the basis of visual information.

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